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ABELIAN GROUPS IN RUSSIA

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Dedicated to the memory of L.Ya. Kulikov

1. The beginning. The Russian mathematicians Kurosh (1908–1971), Mal'cev (1909–1967), Pontryagin (1908–1980), S.V. Fomin (1917–1975), Kaluzhnin (1914–1990) and Lyapin began research on abelian groups in the 1930's. Kurosh [**101**] and Mal'cev [**109**] obtained a characterization of torsion-free abelian groups of finite rank, now known as the Kurosh-Mal'cev-Derry description, see also [**43**]. Kaluzhnin simplified this result in [**65**]. Kurosh [**100**] also found an important description of abelian groups with the minimum condition for subgroups, (see [**43**], Theorem 25.1). Pontryagin proved his now well-known criterion for a countable torsion-free abelian group to be free and constructed an example of an indecomposable torsion-free group of rank 2 [**129**]. Lyapin considered decompositions of a torsion-free finite-rank group into a direct sum of rank-1 groups [**108**]. S.V. Fomin [**41**] investigated the splitting problem for mixed abelian groups (see [**43**, Theorem 100.1]).

All of the mathematicians mentioned above obtained results in other areas of mathematics as well. The first Russian mathematician who devoted his entire mathematical life to abelian groups was L.Ya. Kulikov (1914–2001). His first paper [**99**] was published in 1941, followed by 20 years of considerable activity in the subject. We do not discuss Kulikov's results here because they are classic and most of them are included in the monograph by Fuchs [**43**]. Abelian group theory began to be identified as a separate branch of algebra because of Kulikov's research in the 1940–1960's. This status was confirmed by the publication of the monograph [**42**] by Fuchs in 1958. Those algebraists whose results were included in this book may be viewed as the founders of abelian group theory. Their investigations opened further opportunities for future generations of researchers.

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Mishina began researching abelian groups in 1950. She investigated direct products of rank-1 torsion-free groups: their isomorphisms, direct summands, complete decomposability, separability, and so on. [**114**]. In particular, she described such groups for which every automorphism of any subgroup, respectively factor-group, can be extended to (is induced by) an automorphism of the group and the same for endomorphisms [**115**].

Many other Russian mathematicians have been involved in research on abelian groups since 1960. Myshkin investigated mixed groups of torsion-free rank 1 [**124**–**126**]. Zhuravsky [**160**] obtained some results concerning the splitting of mixed abelian groups and the structure of $\mathrm{Ext}\,(B, A)$ with A torsion and B torsion free. Rudyk [**133**] investigated the splitting problem for the groups for which extensions of automorphisms exist from the torsion subgroup to the whole group.

Kulikov worked at Moscow Pedagogical State University (MPSU) from 1963 on, where he organized and led a permanent algebra seminar. This seminar was very important to the development of abelian group theory in Russia. All of the Russian and many of the foreign results on abelian groups were discussed in this seminar. In addition, two seminars were held at Moscow Lomonosov State University (MSU): the "Abelian Groups" seminar led by Mishina and the closely related "Rings and Modules" seminar led by Skornyakov (1924–1989) and Mikhalev.

Bekker (1928–1997) worked at Tomsk State University (TSU) beginning in 1956. By the end of the 1960's, he had organized a new center of abelian group theory in the algebra department of TSU. Thus there were three centers of abelian groups (MPSU, MSU and TSU) in Russian at the beginning of the 1970's.

Later, Yakovlev, an algebraist whose research interests in algebra were varied, got involved in abelian group theory in the mid 1970's. By the end of the 1970's, he had created the fourth center of abelian groups in St. Petersburg. This group of mathematicians is small but excellent.

A brief description of the four Russian research centers in abelian group theory and a brief summary of some of the accomplishments of their researchers follows below.

2. Abelian groups in MPSU (Moscow). The following Ph.D. students worked in abelian group theory under Kulikov's direct supervision: Kelberger, Gvaramia, Moskalenko, Vedel, Seregina (Deshura), Solonina, Surmanidze, Soifer, A.M. Ivanov, Grishin, A.A. Fomin, Serdukova (Ryatova), Kryuchkov, Rychkov, Farukshin, Tsibikova, Truhmanov, Komarov, Antonova, Degtyarenko and Quan. Kompantseva was a Ph.D. student of Moskalenko. The following Ph.D. students worked under A.A. Fomin's supervision: Cherednikova, Kovyazina, Karpova, Ivantsov, Cheglyakova, Tsarev, Luchishina and Puzakov.

Moskalenko [**119, 120**] and Seregina [**153**] researched nonabelian central extensions of abelian groups with the help of abelian groups themselves. Moskalenko also considered the splitting length of an abelian group [**121**], coperiodic hull of an abelian group [**122**] and multiplications on abelian groups [**123**]. Kompantseva [**71**] investigated subgroups of an abelian group G which were ideals (nil ideals, nilpotent ideals) of every ring with additive group G. In particular, she described the absolute radical of an abelian group. Soifer [**154**] and Lebedenko [**105**] investigated groups which have sets of generators with some fixed properties. Grishin [**55**] considered the multiplicative group of a field.

A.A. Fomin investigated tensor products of torsion-free groups [**28**], torsion-free finite rank groups with various properties [**29**–**32**], dualities in the class of torsion-free groups of finite rank [**33, 34**], categories dual to the category of torsion-free finite-rank abelian group with quasi-homomorphisms [**35, 36**] groups with a prescribed endomorphism (with Mutzbauer) [**37**] and quotient divisible groups (with Wickless) [**38**–**40**].

Cherednikova [**23**] obtained a complete classification of quasi-endomorphisms algebras for torsion-free groups of rank 3. Ivantsov characterized rank-2 Butler groups. Kovyazina [**72**] described dual objects for the Butler groups of arbitrary rank in the sense of the duality [**36**]. Karpova [**67**] generalized Murley's groups. Cheglyakova [**17**] described injective modules over the ring of pseudo-rational numbers. Tsarev investigated torsion-free groups of pseudo-rational rank 1 (see also [**40**]). Luchishina [**107**] and Puzakov considered various generalizations of universal numbers.

Kryuchkov obtained conditions for groups of extensions of locally compact abelian groups to be zero [**94**]. He proved the existence of the following two isomorphisms: $\operatorname{Ext}(K, A) \cong K^* \otimes A$ for an arbitrary compact abelian group K and an arbitrary discrete abelian group A, where K^* is the group of Pontryagin's characters [**95**], and $\mathrm{Hom}\,(K, A) \cong \mathrm{Tor}\,(K^*, A)$ for an arbitrary compact module K and an arbitrary discrete module A in the category of modules over the ring of p-adic integers [**96**]. Kryuchkov also described injective and projective objects in the category of locally compact modules over a ring of integers of a global field [**97**]. He also introduced and investigated slender abelian groups with respect to a fixed topology and with respect to a particular topological group [**98**].

Rychkov (1955–1993) was one of the most outstanding of Kulikov's students. He expanded the concept of a slender group in the sense of Łoš to the concept of a generalized slender mixed abelian group [**134**]. Rychkov also obtained several nice theorems characterizing homomophisms from direct products of groups to generalized slender groups and, dually, from dual slender groups to direct sums of groups [**147**]. Rychkov proved the following generalization of a well-known theorem of Łoš by showing that a quotient group $\prod_i G_i/\sum_i G_i$ is algebraically compact if and only if all but countably many factors are algebraically compact [**135**]. A.M. Ivanov [**56**] obtained this result for cotorsion groups. Rychkov and A.A. Fomin [**149**] described abelian groups which have countably many subgroups. Rychkov investigated the realization of rings as the endomorphism rings of abelian groups [**137, 138**]. He obtained also some deep results in logic [**139**–**141**]. Rychkov applied axiomatics of set theory in his last series of papers [**142**–**146**]. In particular, he proved the insolvability in ZFC of several problems from various areas of abelian group theory. He passed away too soon; many of his ideas remain unrealized.

3. Abelian groups in MSU (Moscow). The main topics of research here were close to those presented in the book [**116**] concerning some generalizations of purity.

Manovtsev [**110**] described all the inductive ω-purities in the category of abelian groups (see the definitions in [**116**]). They were just the intersections of two purities investigated previously by Rohlina and Head. Kuzminov found two sets of integers connected with the functor $\omega\mathrm{Ext}$, which uniquely determined an inductive purity [**104**]. Manovtsev [**111**] found conditions of existence for the following two canonical isomorphisms $\omega\mathrm{Ext}\,(\oplus_{i\in I} C_i, A) \cong \prod_{i\in I} \omega\mathrm{Ext}\,(C_i, A)$ and

$\omega\text{Ext}\,(C, \prod_{i\in I} A_i) \cong \prod_{i\in I} \omega\text{Ext}\,(C, A_i)$.

A.V. Ivanov worked in abelian group theory for a short period of time, but very fruitfully. He solved, or partially solved, the following problems of Fuchs [**43**]: 6b [**57**], 28b [**58**], 44 [**59**], 76 [**60**], 82 [**61**], 84 [**62**], 89 [**63**]. He also considered some problems concerning ω-purities [**64**].

Samsonova [**150**] investigated injective and projective closures of some ω-purities. Komarov [**69, 70**] characterized slender groups in terms of ω-purities. Maximov generalized the Crawly-Jonsson's theorem concerning isomorphic refinements of direct decompositions of a group [**112**]. He also considered near-refinements and quasi-refinements [**113**].

Kravchenko found all completely decomposable torsion-free groups in which every pure (regular) subgroup is completely decomposable [**77**]. For pure subgroups of finite rank the problem had been solved earlier by Kushnir [**102**]. Kravchenko [**78**] solved Warfield's problems 6 and 7 from [Lect. Notes **616** (1977), p. 30]. He also described groups with isomorphic N-high subgroups for each subgroup N. Krivonos [**79**] gave necessary and sufficient conditions for N-high subgroups to be unique in the group.

Kushnir [**103**] characterized finite-rank quotient divisible groups with the help of matrices used by O'Campbell. Vlasova [**155**] obtained some results concerning the determination of groups by their groups of homomorphisms. Kamalov [**66**] partially solved Fuchs's problems 5 and 9.

4. Abelian groups in TSU (Tomsk, Siberia). A number of specialists in abelian group theory appeared at the Algebra Department of Tomsk State University in Siberia at the beginning of the 1970's. They were Bekker (head), Grinshpon, Dobrusin, Kozhuhov, Krylov (present head), Misyakov, Rososhek and Chekhlov. The following students began research on abelian groups at this center during the last 30 years: Eltsova, Konovalov, Mordovskoi, Nikiforov, Pahomova, Podberezina, Prihodovsky, Sebeldin, Timoshenko, Turmanov, Faustova, Flesher, Friger, Hayut, Shaposhnikov, Shaposhnikova, Sherstneva and Shlafer.

The Algebra Department of TSU published 15 volumes of the annual journal "Abelian Groups and Modules." The main subjects of research were algebraically compact groups and related groups, endomorphism rings and fully invariant subgroups, automorphism groups, the groups Hom and Ext, holomorphs and first cohomology groups.

4.1. Algebraically compact and related groups. The class of algebraically compact groups coincides with the class of pure injective groups. Dobrusin considered the following generalization of this concept. A group A is called quasi-pure injective, a QPI-group for short, if every homomorphism $G \to A$ can be extended to an endomorphism of A for each pure subgroup G of A. He described QPI-groups in a class of torsion-free reduced abelian groups with some restrictions on the types of elements [**24**]. Chekhlov completed this description later for arbitrary torsion-free abelian groups [**19, 20**]. If, in the definition of QPI-groups, the subgroups G are closed in the Z-adic topology, then we obtain the definition of a QCPI-group. Chekhlov investigated QCPI-groups [**22**] and the closely related CS-groups [**18**], where a group is a CS-group if the closure in the Z-adic topology of each of its pure subgroups is a direct summand.

A reduced group is called (fully) transitive if, for every two elements a and b, the (inequality) equality of height-matrices $(H(a) \leq H(b))H(a) = H(b)$ implies the existence of an (endomorphism) automorphism of the group mapping a to b. Chekhlov proved that every torsion-free QPI-group is transitive [**20**]. Dobrusin considered fully transitive and transitive groups in the classes of completely decomposable groups, vector groups, separable groups and finite-rank torsion-free groups [**25**]. Krylov described the structure of a countable homogeneous fully transitive group without torsion [**81**]. Such a group is determined by its type and by the topological ring of endomorphisms. He also considered nonhomogeneous fully transitive groups with a special pseudo-socle [**82**]. Chekhlov characterized fully transitive torsion-free groups, which have only injective nonzero endomorphisms, and also quasi-homogeneous fully transitive groups of finite p-rank, where p is a prime [**21**]. Grinshpon described homogeneously decomposable fully transitive groups without torsion [**46, 47**].

Grinshpon also investigated torsion and mixed fully transitive groups [**48, 54**]. He proved that p-groups with a cyclic first Ulm's subgroup were fully transitive [**48**]. He also found some criterions for the full transitivity of K-direct (see the definition in [**43**, vol. 1]) sums of groups for some classes [**50, 54**]. In particular, he proved that every K-direct sum of algebraically compact groups is fully transitive. Misyakov found out that the direct product of torsion groups is fully transitive if and only if the direct sum was fully transitive [**118**].

4.2. Endomorphism rings and fully invariant subgroups. Krylov solved the principal problems about the Jacobson radical $J(E(A))$ of the endomorphism ring $E(A)$ for a torsion-free group A of finite rank [**80**]. He characterized torsion-free groups of finite rank which were flat or generating modules over their endormorphism rings [**83, 84**]. Krylov also considered groups and modules with hereditary endomorphism rings and obtained analogs of the Baer and Baer-Kolettis theorems for completely decomposable groups [**85, 86**]. He considered groups admitting a natural embedding into the direct product of their p-components as a pure subgroup, their endomorphism rings, radicals and these groups as modules over their endomorphism rings [**90, 92**].

Podberezina and Pahomova researched groups $\mathrm{Hom}\,(A,B)$ and $A\otimes B$ as Artinian, Noetherian or injective modules over the rings $E(A)$ or $E(B)$ [**128, 93**]. The characterization problem was completely solved for an Artinian or injective $E(B)$-module $\mathrm{Hom}\,(A,B)$. Rososhek and Turmanov solved some problems concerning purity in the sense of Cohn [**132**]. Bekker and Rososhek considered groups as modules over one prescribed endomorphism [**8**].

Grinshpon called a reduced group A an H-group if each of its fully invariant subgroups is of the form $A(M)=\{a\in A \mid H(a)\geq M\}$, where M is an $\omega{\times}\omega$-matrix with cardinals and symbols ∞ as entries. He found necessary and sufficient conditions to be an H-group for: a) K-direct sums of torsion groups; b) K-direct sums of homogeneous torsion-free groups; c) p-local groups; d) finite-rank torsion-free groups [**46, 50, 54**]. He described fully invariant subgroups and their lattices for separable p-groups, separable torsion-free groups and vector groups [**49, 53**]. Sebeldin investigated the problem of determination of an abelian group by its endomorphism ring (group) in various classes [**151, 152**].

4.3. Groups of homomorphisms, extensions, automorphisms, first cohomology groups and holomorphs. Krylov found conditions for a torsion-free group A and an arbitrary group B such that $\mathrm{Ext}\,(A, C)$ is isomorphic to $\mathrm{Ext}\,(B, C)$ or $\mathrm{Ext}\,(C, A)$ is isomorphic to $\mathrm{Ext}\,(C, B)$ for each group C. This partially solved Problem 43 of Fuchs [**42**]. He also found some conditions for groups A and B so that $\mathrm{Ext}\,(A, B)$ and $\mathrm{Ext}\,(B, A)$ were simultaneously torsion-free. In particular, he answered Question 11.51 of Mader in the Kourovka notebook concerning the existence of large nontrivial classes of torsion-free abelian groups for which $\mathrm{Ext}\,(A, B)$ is torsion-free for each pair of groups A and B in this class [**87**–**89**].

Grinshpon found conditions for the groups A and B for which $\mathrm{Hom}\,(A, B)$ is equal to zero for a torsion or separable torsion-free group B [**51**]. He also found necessary and sufficient conditions for p-groups A and B for which an isomorphism of endomorphism groups $\mathrm{End}\,A \cong \mathrm{End}\,B$ implies $A \cong B$ [**45**]. This solved Fuchs's Problem 41 from [**42**].

Kozhuhov investigated automorphisms of torsion-free groups. He described completely decomposable torsion-free groups which have a regular automorphism. He also proved that if the regular automorphisms of a finite rank group with the identity formed a subgroup, then this subgroup coincided with the whole group of automorphisms or with the identity. He found a method to investigate torsion-free finite-rank groups with finite groups of automorphisms. Many problems can be reduced to a solution of systems of congruences of the first degree [**4, 73**–**76**].

Kozhuhov and Nikiforov described rank-2 torsion-free groups with a cyclic group of automorphisms of order 4 and 6 [**76**] (see also the analogous result of A.A. Fomin and Mutzbauer [**37**]). Faustova considered torsion-free groups of an arbitrary finite rank which have an automorphism of order 4 or 6 [**27**].

Bekker applied abelian group theory to first cohomology groups and holomorphs. He solved the problem of determining an abelian group by its holomorph. He also proved that every nonreduced torsion-free group is characteristic in its holomorph and that the holomorph of every torsion reduced group without elements of order 2 is complete. He obtained some conditions for the first cohomology groups to be equal to zero over abelian groups of various classes [**1**–**4, 6**]. In the last period of his life Bekker investigated the affine groups of modules, a generalization of the holomorph [**7**].

5. Abelian groups in SPSU (St. Petersburg). A considerable contribution to abelian group theory was made by the mathematicians of St. Petersburg State University during the last 25 years, especially concerning classification and decomposition problems for torsion-free abelian groups. In 1976 Yakovlev, using a new matrix representation of finite rank groups, showed that the classification problem is of "wild" type [**156**].

In the mid 1980's his student, Blagoveshchenskaya, found a new graphical approach to direct decompositions of finite rank torsion-free abelian groups [**9**], which led to the solutions of two problems of Fuchs's monograph [**43**], namely Problem 67 [**10**] and Problem 68 [**11**] by Blagoveshchenskaya and Yakovlev. At first she considered decomposition problems for almost completely decomposable groups with cyclic regulator quotient (crq-groups) [**9**]. Various decompositions of such a group were described as special transformations of a certain graph whose connected components corresponded to indecomposable summands of the group. Later Blagoveskhchenskaya and Yakovlev [**11**] extended these results to all torsion-free abelian groups of finite rank. The graphical representation of block-rigid crq-groups was generalized by Blagoveshchenskaya and Mader, a complete classification of these groups up to near-isomorphism was obtained [**12**]. The results were extended by Blagoveshchenskaya and Goebel to some Butler groups of countable rank [**15**]. Blagoveshchenskaya, G. Ivanov and Schultz proved a version of the Baer-Kaplansky theorem for block-rigid crq-groups of ring type. The groups of this class turned out to be determined by their endomorphism rings up to near-isomorphism [**14**].

Yakovlev [**158**] reduced consideration of finite rank torsion-free abelian groups to vectors in a cone of an integral lattice. This opened new prospects for his students. Kamara N'Famara found a class of mixed groups that admitted an analogous representation. Lebedinsky [**106**] extended this approach to some "appropriate" groups but, instead of the rank in the usual sense he considered a "generalized" rank (the number of indecomposable summands of the group, considered as an object in a special category). Blazhenov gave a satisfactory classification of the isomorphism classes in a genus class and applied the result to the study of various aspects of the cancellation of modules [**16**].

This yielded solutions of Fuchs's Problems 70 and 71. An extension of the decomposition theory from torsion-free abelian groups to some Artinian modules was made by Yakovlev [**159**]. Together with his student Pimenov he found examples of modules for which the Krull-Schmidt theorem failed.

Generalov considered the following generalization of purity. Let $\mathcal{M}$ be a class of R-modules. The proper class flatly generated by $\mathcal{M}$ consists of all monomorphisms $i : A \to B$ in the category Mod-R such that all induced homomorphisms $i \otimes id_M : A \otimes M \to B \otimes M$ are injective. Generalov extended the "Manovtsev-Kuzminov theorem," which holds for the category of abelian groups, to the category of modules over a bounded Dedekind prime ring. In fact, he proved a more general result over hereditary Noetherian prime rings (HNP-rings): specifically, if R is a bounded HNP-ring, then every inductively closed proper class in Mod-R is flatly generated if and only if R is a Dedekind prime ring [**44**]. Other generalizations concerned the description of algebraically compact abelian groups given in Kaplansky's theorem [**43**, Theorem 40.1]. They were proved in the cases where R was either a bounded HNP-ring or a tame hereditary finite-dimensional algebra over a field [**44**].

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